

**Macao**

## Introduction to RAISE

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**RAISE?**

Radical **A**lternative for **I**nadequate **S**oftware **E**ngineers  
Rambling **A**round **I**n **S**earch of **E**nlightenment

## Rigorous Approach to Industrial Software Engineering

RAISE is a product consisting of:

- a method for software development
- a formal specification language: RSL
- computer based tools

developed by:

- DDC/CRI (DK)
- STL/BNR (UK)
- ICL (UK)
- NBB/ABB/SYPRO (DK)

in an ESPRIT-I project, RAISE, 1985 - 1990

## Background

model-oriented (VDM, Z, ...)

property-oriented (Clear, ...)

concurrency (CSP, ...)

structuring (ML, ...)

tools

RAISE

## RAISE Continuation

ESPRIT-II project, LaCoS, 1990 - 1995

Large Scale Correct Systems

Using Formal Methods

- industrial applications of RAISE
- evolution of RAISE method, language and tools

## LaCoS Partners

### Producers:

- CRI (DK)
- SYPRO (DK)
- BNR Europe (UK)

### Consumers:

- BNR Europe (UK): Network design toolset
- Lloyd's Register (UK): Ship engine monitoring; security
- Bull (F): Database; security
- MATRA Transport (F): Automatic train protection
- Inisel Espacio (E): Image processing
- Space Software Italia (I): Tethered satellite; air traffic control
- Technisystems (GR): Shipping transaction processing

## RAISE Specification Language (RSL)

Design objectives:

- Wide spectrum language
  - Abstract — concrete; specification — implementation in one language
  - Applicative and imperative styles
  - Sequential and concurrent styles
  - Maximal applicability (but better for information systems than control systems: time added later)
- Suitable for large descriptions; modular

## Design objectives: type system

- Type checking simple:
  - decidable
  - minimal type inference required
  - separation of types and values (sets are not types)

## Design objectives: user friendly

- User convenience preferred to tool writers' convenience:
  - No “define before use” restriction
  - Language tightly defined, nothing “implementation dependent” (such as evaluation order)
- Expressions have maximum expressivity; modular concepts are minimal
- Implementation relation is simple: just property preservation; no fitting morphisms
- Powerful logic: implementation relation can be expressed in RSL

## Regularity

- Maximum reuse of binding, typing, pattern, etc.
- When a construct (expression, type, binding, etc.) is allowed, *any* form of the construct should be allowed.

## Portability

ASCII syntax:

| Sym           | ASCII  | Sym                  | ASCII | Sym                  | ASCII    | Sym               | ASCII |
|---------------|--------|----------------------|-------|----------------------|----------|-------------------|-------|
| $\times$      | ><     | *                    | -list | $\omega$             | -inflist | $\rightarrow$     | ->    |
| $\rightarrow$ | -~->   | $\overrightarrow{m}$ | -m->  | $\overrightarrow{m}$ | -~m->    | $\leftrightarrow$ | <->   |
| $\wedge$      | \      | $\vee$               | \     | $\Rightarrow$        | =>       | $\forall$         | all   |
| $\exists$     | exists | $\bullet$            | :-    | $\square$            | always   | $\equiv$          | is    |
| $\neq$        | ~=     | $\leq$               | <=    | $\geq$               | >=       | $\uparrow$        | **    |
| $\in$         | isin   | $\notin$             | ~isin | $\subset$            | <<       | $\subseteq$       | <<=   |
| $\supset$     | >>     | $\supseteq$          | >>=   | $\cup$               | union    | $\cap$            | inter |
| $\dagger$     | !!     | $\langle$            | <.    | $\rangle$            | .>       | $\mapsto$         | +>    |
| $\parallel$   |        | $\#$                 | ++    | $\square$            | =        | $\Pi$             | ^     |
|               |        | $\lambda$            | -\    | $\circ$              | #        |                   |       |

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13

## Conventions for tools

- Files have `.rs1` suffix
- One module per file
- Module name same as file base name

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14

## UNU-IIST RAISE tools

- Open source; Gnu Public Licence
- Written (effectively) in C, so very portable
- Command line tool using emacs to provide interface: aids portability

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15

## UNU-IIST RAISE tools: capabilities

- Type checking
- Pretty-printing
- Module dependency display (graph or table)
- Confidence condition generation
- Translation to SML and C++
- Translation to PVS for proofs
- Generation from UML class diagrams

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16

## Design achievements

- Unification of algebraic and model-based approaches
- Unification of channel-based concurrency with value passing

## A simple example

```
scheme REGISTRATION =  
  class  
    type  
      Database = Person-set,  
      Person = Text  
    value  
      empty : Database = {},  
  
      register : Person × Database → Database  
      register(p,db) ≡ db ∪ {p},  
  
      is_registered : Person × Database → Bool  
      is_registered(p,db) ≡ p ∈ db  
  end
```

## Questions about REGISTRATION

- What happens if someone registers twice?
- What happens if two people have the same name?
- Could you use this to register the people for this course?
- Could you use it to register the people of China?

## Another example

```
scheme SJH =  
  class  
    type  
      Sdfv = Jdmjh-set,  
      Jdmjh = Text  
    value  
      dfm : Sdfv = {},  
  
      mjd : Jdmjh × Sdfv → Sdfv  
      mjd(mn,dfmn) ≡ dfmn ∪ {mn},  
  
      mjdwr : Jdmjh × Sdfv → Bool  
      mjdwr(mn,dfmn) ≡ mn ∈ dfmn  
  end
```

## Characteristics of specifications

The two examples are isomorphic. To most mathematicians, this means they are the same.

Aims of specification (ordered):

1. Capture requirements precisely and clearly
2. Support the exploration of requirements; the raising of questions
42. Provide a basis for implementation

Specifications need *interpretation*, a relation between their types, values, modules, etc. and the real world.

## Types in RSL

### Types and functions in RSL

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Types may be **abstract** or **concrete** (and the two may be mixed)

**type**

```
Database = Key  $\overline{m}$  Data,  
Key,  
Data
```

Key and Data are abstract types: no definitions. Database is concrete — it is defined as the finite mapping (many-one relation) from Key to Data. Database could also be abstract; Key and Data could be concrete.

Both concrete and abstract types come with a built-in equality relation on their values.

## Built-in types

- **Bool**
- **Int**
- **Nat** (=  $\{ | i : \text{Int} \cdot i \geq 0 | \}$ )
- **Real**
- **Char**
- **Text** (= **Char**<sup>\*</sup>)
- **Unit**

|                 |   |
|-----------------|---|
| <b>Bool</b>     | values: <b>true, false</b><br>connectives: $\wedge, \vee, \Rightarrow, \sim$  |
| <b>Int, Nat</b> | values: ..., -2, -1, 0, 1, 2, ...<br>operators: +, -, *, /, $\uparrow, \backslash, <, \leq, >, \geq$ , <b>abs, real</b> |
| <b>Real</b>     | values: ..., -4.3, ..., 0.0, ..., 1.0, ...<br>operators: +, -, *, /, $\uparrow, <, \leq, >, \geq$ , <b>abs, int</b>     |
| <b>Char</b>     | values: 'a', ...  |
| <b>Text</b>     | values: "Alice", ...<br>operators: <b>hd, tl, <math>\hat{\phantom{a}}, \_(-)</math>, len, inds, elems</b>               |
| <b>Unit</b>     | value: ()   |

## Type constructors

Product:  $T \times U, T \times U \times V, \dots$   $(t,u), (t,u,v)$   
 Set: **T-set, T-infset**  $\{\}, \{t_1,t_2\}$   
 List:  $T^*, T^\omega$   $\langle \rangle, \langle t_1,t_2 \rangle$   
 Map:  $T \xrightarrow{m} U, T \xrightarrow{\tilde{m}} U$   $[], [t_1 \mapsto u_1, t_2 \mapsto u_2]$   
 Function:  $T \rightarrow U, T \xrightarrow{\sim} U$   $\lambda x:T \cdot u(x)$

Integer sets and lists have ranged values, such as  $\{0..10\}$ , and  $\langle 1..12 \rangle$ .

Sets, lists, and maps have comprehended values, such as  $\{2*x+1 \mid x : \text{Int} \cdot x \in \{0..4\}\}$ ,  $\langle x \mid x \text{ in } \langle 0..10 \rangle \cdot \text{is\_odd}(x) \rangle$ , and  $[f(x) \mapsto g(x) \mid x : \text{Int} \cdot p(x)]$

## Products

A product is  
 an ordered finite collection  
 of  
 values of possibly different types

Examples:

$(1,2)$   
 $(1, \text{true}, \text{"John"})$

## Product Type Expressions

$\text{type\_expr}_1 \times \dots \times \text{type\_expr}_n, n \geq 2$

Values:

$(v_1, \dots, v_n), v_i : \text{type\_expr}_i$

Operators:

$=$   
 $\neq$

## Example: A System of Coordinates I

```
SYSTEM_OF_COORDINATES =
class
  type
    Position = Real × Real
  value
    origin : Position = (0.0,0.0),

    distance : Position × Position → Real
    distance((x1,y1),(x2,y2)) ≡
      ((x2-x1)↑2.0 + (y2-y1)↑2.0)↑0.5
end
```



## Example: A System of Coordinates II

```

SYSTEM_OF_COORDINATES =
class
  type Position = Real × Real
  value
    origin : Position = (0.0,0.0),
    distance : Position × Position → Real
    distance(p1, p2) ≡
      let
        (x1,y1) = p1,
        (x2,y2) = p2
      in ((x2-x1)↑2.0 + (y2-y1)↑2.0)↑0.5
      end
end

```

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9

## Records: example 1

```

SYSTEM_OF_COORDINATES =
class
  type
    Position ::
      x_coord : Real
      y_coord : Real
  value
    origin : Position = mk_Position(0.0,0.0),
    distance : Position × Position → Real
    distance(p1, p2) ≡
      ((x_coord(p2) - x_coord(p1))↑2.0 +
      (y_coord(p2) - y_coord(p1))↑2.0)↑0.5
      end
end

```

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10

## Records: example 2

```

type
  Book ::
    title: Title
    author : Author
    date : YearMonth
    price : Real ↔ change_price,
  YearMonth ::
    year : Year
    month : Month,
  Month = { | n : Nat • n ∈ {1..12} | }

```

mk\_Book is a **constructor** of type  $\text{Title} \times \dots \times \text{Real} \rightarrow \text{Book}$   
 title is a **destructor** of type  $\text{Book} \rightarrow \text{Title}$   
 change\_price is a **reconstructor** of type  $\text{Real} \times \text{Book} \rightarrow \text{Book}$

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11

## Variants

```

type
  Cowboy == good | bad | ugly,
  OptId == no_id | an_id(id : Id),
  Tree == nil | node(left : Tree, val : Val, right : Tree)

```

good, bad, ugly, no\_id, an\_id, nil, and node are **constructors**  
 id, left, val, and right are **destructors**.

nil has type Tree  
 node has type  $\text{Tree} \times \text{Val} \times \text{Tree} \rightarrow \text{Tree}$   
 val has type  $\text{Tree} \rightarrow \text{Val}$

Only variant type definitions may be recursive.

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12

## Case expressions

Lists and variants are often analysed by case expressions, as in:

```
value
reverse : T* → T*
reverse(t) ≡
  case t of
    ⟨⟩ → ⟨⟩,
    ⟨h⟩^t → reverse(t)^⟨h⟩
  end,
traverse : Tree → Val*
traverse(t) ≡
  case t of
    nil → ⟨⟩,
    node(l, v, t) → traverse(l) ^ ⟨v⟩ ^ traverse(t)
  end
```

## Partial functions: example

```
value
factorial : Nat  $\overset{\sim}{\rightarrow}$  Nat
factorial(n) ≡
  if n = 1 then 1 else n * factorial(n - 1) end
pre n > 0
```

A partial function has  $\overset{\sim}{\rightarrow}$  in its signature and **pre** in its definition.

## Implicit functions: example

```
value
square_root : Real  $\overset{\sim}{\rightarrow}$  Real
square_root(x) as r post r * r = x
pre x ≥ 0.0
```

An implicit function uses **post**, usually with **as**, in its definition.

## A better square\_root specification?

```
value
square_root : Real  $\overset{\sim}{\rightarrow}$  Real
square_root(x) as r post r * r = x ∧ r ≥ 0.0
pre x ≥ 0.0
```

## What about this specification?

### An even better square\_root specification?

value

```
square_root : Real × Real  $\rightsquigarrow$  Real
square_root(x,  $\epsilon$ ) as r post abs(r * r - x)  $\leq$   $\epsilon$   $\wedge$  r  $\geq$  0.0
pre x  $\geq$  0.0  $\wedge$   $\epsilon$  > 0.0
```

value

```
square_root : Real × Real  $\rightsquigarrow$  Real
square_root(x,  $\epsilon$ )  $\equiv$ 
  if x = 0.0 then 0.0 else newton_raphson(x,  $\epsilon$ , x/2.0) end
pre x  $\geq$  0.0  $\wedge$   $\epsilon$  > 0.0,

newton_raphson : Real × Real × Real  $\rightsquigarrow$  Real
newton_raphson(x,  $\epsilon$ , r)  $\equiv$ 
  if abs(r * r - x)  $\leq$   $\epsilon$  then r
  else newton_raphson(x,  $\epsilon$ , (r + x/r)/2.0) end
pre x  $\geq$  0.0  $\wedge$   $\epsilon$  > 0.0  $\wedge$  r > 0.0
```

### One more version

value

```
newton_raphson : Real × Real × (Real  $\rightsquigarrow$  Real) × (Real  $\rightsquigarrow$  Real)  $\rightsquigarrow$  Real
newton_raphson(r,  $\epsilon$ , f, f')  $\equiv$ 
  if abs(f(r))  $\leq$   $\epsilon$  then r
  else
    let r1 = r - f(r) / f'(r) in
      newton_raphson(r1,  $\epsilon$ , f, f')
  end
end
pre  $\epsilon$  > 0.0  $\wedge$  f'(r)  $\neq$  0.0,
```

value

```
square_root : Real × Real  $\rightsquigarrow$  Real
square_root(x,  $\epsilon$ )  $\equiv$ 
  if x = 0.0 then 0.0
  else
    let
      f =  $\lambda$  a : Real • a * a - x,
      f' =  $\lambda$  a : Real • 2.0 * a
    in newton_raphson(x - f(x)/f'(x),  $\epsilon$ , f, f')
  end
end
pre x  $\geq$  0.0  $\wedge$   $\epsilon$  > 0.0
```

## Quiz 1

```

value
cube_root : Real × Real  $\rightsquigarrow$  Real
cube_root(x,  $\epsilon$ )  $\equiv$ 
  if x = 0.0 then 0.0
  else
    let
      f =  $\lambda$  a : Real • a * a * a - x,
      f' =  $\lambda$  a : Real • 3.0 * a * a
    in newton_raphson(x - f(x)/f'(x),  $\epsilon$ , f, f')
    end
  end
pre x  $\geq$  0.0  $\wedge$   $\epsilon$  > 0.0

```

Letters could be Roman letters, Arabic letters, etc. There is a special letter *nil* used to indicate the end of a word. Words are lists of letters that satisfy *is\_wf\_Word*:

```

type Word = { | w : Letter* • is_wf_Word(w) | }

```

value

```

is_wf_Word : Letter*  $\rightarrow$  Bool
is_wf_Word(w)  $\equiv$ 
  len w > 0  $\wedge$  w(len w) = nil  $\wedge$ 
  ( $\forall$  i : Nat • i  $\geq$  1  $\wedge$  i < len w  $\Rightarrow$  w(i)  $\neq$  nil)

```

Lists are indexed from 1 in RSL:  $w(1)$  is the first element of the list  $w$ .

1. Is the list  $\langle nil \rangle$  a word?
2. How many nils can there be in a word?

## Quiz 2

What is the logical error in the following?

value

```

/* check first n letters are the same */
match_n : Word × Word × Nat  $\rightsquigarrow$  Bool
match_n(w1, w2, n)  $\equiv$  first_n(w1, n) = first_n(w2, n)
pre n  $\leq$  len w1  $\wedge$  n  $\leq$  len w2,

```

```

/* select the first n letters of a word */
first_n : Word × Nat  $\rightsquigarrow$  Word
first_n(w, n)  $\equiv$ 
  if n = 0 then  $\langle \rangle$  else  $\langle$ hd w $\rangle$  ^ first_n(tl w, n-1) end
pre n  $\leq$  len w

```

**hd**  $w$  gives the head (first element) of a list  $w$ , and **tl**  $w$  gives the tail (the list  $w$  with its head removed).

Hints:

- Read the code carefully
- Try checking confidence conditions
- Try some test cases. Try setting Letter to Char, nil to '0', and execute

test\_case

```

[t1] first_n("abc0", 1),
[t2] first_n("abc0", 2),
[t3] first_n("abc0", 3),
[t4] first_n("abc0", 4)

```

Use the SML translator and the C++ translator. Any differences?

## Exercises

These exercises are based on the type `Tree` defined by

```
type Tree == nil | node(left : Tree, val : Int, right : Tree)
```

1. Define a function 'depth' that returns the depth of a tree.
2. Define a function 'is\_in' to find if an integer is in a tree.
3. Define the subtype 'Ordered\_tree'. The subtype should not allow repetitions, so that an ordered tree models a set.
4. Define a function 'is\_in\_ordered' to find if an integer is in an ordered tree.
5. Define a total function 'add' to add an integer to an ordered tree.
6. Define a total function 'remove' to remove an integer from an ordered tree.

## Sets, lists, and maps

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### Sets

- finite and infinite sets
- set type expressions
- set operators
- set value expressions
- example of specification using sets

### Sets

A set is:

an unordered collection  
of  
values of same type

Examples:

$\{1, 3, 5\}$   
 $\{\text{"John"}, \text{"Peter"}, \text{"Ann"}\}$

### Set Type Expressions

- **type\_expr-set**  
 $\{v_1, \dots, v_n\}$   
where  $n \geq 0, v_i : \text{type\_expr}$
- **type\_expr-infset**  
 $\{v_1, \dots, v_n\},$   
 $\{v_1, \dots, v_n, \dots\}$   
where  $n \geq 0, v_i : \text{type\_expr}$

## Associated Built-in Operators

|  |  |
|--|--|
| $\cup : \mathbf{T-infset} \times \mathbf{T-infset} \rightarrow \mathbf{T-infset}$      | $\{1, 2, 3\} \cup \{3, 4\} = \{1, 2, 3, 4\}$ |
| $\cap : \mathbf{T-infset} \times \mathbf{T-infset} \rightarrow \mathbf{T-infset}$      | $\{1, 2, 3\} \cap \{3, 4\} = \{3\}$          |
| $\setminus : \mathbf{T-infset} \times \mathbf{T-infset} \rightarrow \mathbf{T-infset}$ | $\{1, 2, 3\} \setminus \{3, 4\} = \{1, 2\}$  |
| $\in : \mathbf{T} \times \mathbf{T-infset} \rightarrow \mathbf{Bool}$                  | $4 \in \{1, 2, 3\} = \mathbf{false}$         |
| $\notin : \mathbf{T} \times \mathbf{T-infset} \rightarrow \mathbf{Bool}$               | $4 \notin \{1, 2, 3\} = \mathbf{true}$       |

$\subset : \mathbf{T-infset} \times \mathbf{T-infset} \rightarrow \mathbf{Bool}$

$\{1, 3\} \subset \{1, 2, 3\} = \mathbf{true}$   
 $\{1, 2, 3\} \subset \{1, 2, 3\} = \mathbf{false}$

$\subseteq : \mathbf{T-infset} \times \mathbf{T-infset} \rightarrow \mathbf{Bool}$

$\{1, 3\} \subseteq \{1, 2, 3\} = \mathbf{true}$   
 $\{1, 2, 3\} \subseteq \{1, 2, 3\} = \mathbf{true}$   
 $\{1, 2, 3\} \subseteq \{1, 3\} = \mathbf{false}$

$\supset$  and  $\supseteq$  are similar

$\mathbf{card} : \mathbf{T-infset} \xrightarrow{\sim} \mathbf{Nat}$

$\mathbf{card} \{1, 2, 5, 2, 2, 1, 5\} = 3$   
 $\mathbf{card} \{n \mid n : \mathbf{Nat}\} \equiv \mathbf{chaos}$

## Overloading of hd

Theory:

$\mathbf{hd} : \mathbf{T-infset} \xrightarrow{\sim} \mathbf{T}$   
 $\mathbf{hd}(s) \text{ as } x \text{ post } x \in s$   
 $\mathbf{pre } s \neq \{\}$

Example:

$\mathbf{hd} \{1,2\} \in \{1,2\}$

i.e.

$\mathbf{hd} \{1,2\} = 1 \vee \mathbf{hd} \{1,2\} = 2$

**NB** The overloading of  $\mathbf{hd}$  for sets was added after the RSL book and method book were written.

## Set Value Expressions

Enumerated:  $\{\text{expr}_1, \dots, \text{expr}_n\}$

$\{1,2\}$   
 $\{1,2,1\}$

Ranged:  $\{\text{integer-expr}_1 \dots \text{integer-expr}_2\}$

$\{3 \dots 7\} = \{3,4,5,6,7\}$   
 $\{3 \dots 3\} = \{3\}$   
 $\{3 \dots 2\} = \{\}$

Comprehended:  $\{\text{expr}_1 \mid \text{typing}_1, \dots, \text{typing}_n \bullet \text{logical-expr}_2\}$

$\{2*n \mid n : \mathbf{Nat} \bullet n \leq 3\}$

RESOURCE\_MANAGER =

**class**

**type**

Resource,  
Pool = Resource-**set**

**value**

obtain : Pool  $\rightsquigarrow$  Pool  $\times$  Resource  
obtain(p) **as** (p<sub>1</sub>, r<sub>1</sub>) **post** r<sub>1</sub> ∈ p ∧ p<sub>1</sub> = p \ {r<sub>1</sub>}  
**pre** p ≠ {},

release : Resource  $\times$  Pool  $\rightsquigarrow$  Pool  
release(r,p) ≡ p ∪ {r}

**pre** r ∉ p

**end**

## Lists

- finite and infinite lists
- list type expressions
- list value expressions
- list indexing
- list operators
- example of specification using lists

## Lists

A list is:

an ordered collection  
of  
values of same type

Examples:

⟨1,3,3,1,5⟩  
⟨**true**,**false**,**true**⟩

## List Type Expressions

- type\_expr\*  
⟨v<sub>1</sub>, ..., v<sub>n</sub>⟩  
where n ≥ 0, v<sub>i</sub> : type\_expr
- type\_expr<sup>ω</sup>  
⟨v<sub>1</sub>, ..., v<sub>n</sub>⟩,  
⟨v<sub>1</sub>, ..., v<sub>n</sub>, ...⟩  
where n ≥ 0, v<sub>i</sub> : type\_expr



## List Value Expressions

Enumerated:  $\langle \text{expr}_1, \dots, \text{expr}_n \rangle$

$\langle 1, 3, 3, 1, 5 \rangle$   
 $\langle \text{true}, \text{false}, \text{true} \rangle$

Ranged:  $\langle \text{integer-expr}_1 .. \text{integer-expr}_2 \rangle$

$\langle 3 .. 7 \rangle = \langle 3, 4, 5, 6, 7 \rangle$   
 $\langle 3 .. 3 \rangle = \langle 3 \rangle$   
 $\langle 3 .. 2 \rangle = \langle \rangle$

Comprehended:  $\langle \text{expr}_1 \mid \text{binding in } \text{list-expr}_2 \bullet \text{logical-expr}_3 \rangle$

$\langle 2 * n \mid n \text{ in } \langle 0 .. 3 \rangle \rangle$   
 $\langle n \mid n \text{ in } \langle 0 .. 100 \rangle \bullet \text{is\_even}(n) \rangle$

## List Indexing

Basic form:

$\text{list-expr}(\text{integer-expr}_1)$

Example:

$\langle 2, 5, 3 \rangle(2) = 5$

## Associated Built-in Operators

|  |   |
|--|---|
| $\wedge : T^* \times T^\omega \rightarrow T^\omega$    | $\langle e_1, \dots, e_n \rangle \wedge \langle e_{n+1}, \dots \rangle = \langle e_1, \dots, e_n, e_{n+1}, \dots \rangle$                           |
| <b>hd</b> : $T^\omega \rightarrow T$                   | <b>hd</b> $\langle e_1, e_2, \dots \rangle = e_1$   |
| <b>tl</b> : $T^\omega \rightarrow T^\omega$            | <b>tl</b> $\langle e_1, e_2, \dots \rangle = \langle e_2, \dots \rangle$  |
| <b>len</b> : $T^\omega \rightarrow \text{Nat}$         | <b>len</b> $\langle e_1, \dots, e_n \rangle = n$<br><b>len</b> $\text{il} \equiv \text{chaos}$  |
| <b>elems</b> : $T^\omega \rightarrow T\text{-infset}$  | <b>elems</b> $\langle e_1, e_2, \dots \rangle = \{e_1, e_2, \dots\}$  |
| <b>inds</b> : $T^\omega \rightarrow \text{Nat-infset}$ | <b>inds</b> $\text{fl} = \{1 .. \text{len fl}\}$<br><b>inds</b> $\text{il} = \{\text{idx} \mid \text{idx} : \text{Nat} \bullet \text{idx} \geq 1\}$ |

## Overloading of $\in$ and $\notin$

|  |   |
|--|---|
| $\in : T \times T^\omega \rightarrow \text{Bool}$    | $'d' \in \langle 'a', 'b', 'c' \rangle = \text{false}$    |
| $\notin : T \times T^\omega \rightarrow \text{Bool}$ | $'a' \notin \langle 'a', 'b', 'c' \rangle = \text{false}$ |

**NB** The overloading of  $\in$  and  $\notin$  for lists (and maps) was added after the RSL book and method book were written.

```

QUEUE =
  class
    type
      Element,
      Queue = Element*
    value
      empty : Queue = ⟨⟩,

      enq : Element × Queue → Queue
      enq(e,q) ≡ q ^ ⟨e⟩,

      deq : Queue → Queue × Element
      deq(q) ≡ (tl q,hd q)
      pre q ≠ empty
    end
  end

```

```

SORTING = class
  value
    sort : Int* → Int*
    sort(l) as l1 post is_permutation(l1,l) ∧ is_sorted(l1)

    is_permutation : Int* × Int* → Bool,
    is_permutation(l1,l2) ≡ (∀ i : Int • count(i, l1) = count(i, l2)),

    count : Int × Int* → Nat
    count(i, l) ≡ card {idx | idx : Nat • idx ∈ inds l ∧ l(idx) = i},

    is_sorted : Int* → Bool
    is_sorted(l) ≡
      (∀ idx1,idx2 : Nat • {idx1,idx2} ⊆ inds l ∧ idx1 < idx2 ⇒ l(idx1) ≤ l(idx2))
  end
end

```

## Maps

- map type expressions
- map value expressions
- map application
- map operators
- example of specification using maps

## Maps

A map is:

an unordered collection  
of  
pairs of values

Examples:

```

["Klaus" ↦ 7, "John" ↦ 2, "Mary" ↦ 7]
[1 ↦ 2, 5 ↦ 10]

```

## Map Type Expressions

Maps may be:

- infinite
- partial
- non-deterministic

- $\text{type\_expr}_1 \xrightarrow{m} \text{type\_expr}_2$   
[ $v_1 \mapsto w_1, \dots, v_n \mapsto w_n$ ]  
where  $n \geq 0$ ,  $v_i : \text{type\_expr}_1$ ,  $w_i : \text{type\_expr}_2$   
and  $v_i = v_j \Rightarrow w_i = w_j$   
Finite and deterministic when applied to elements in the domain
- $\text{type\_expr}_1 \xrightarrow{\sim m} \text{type\_expr}_2$   
[ $v_1 \mapsto w_1, \dots, v_n \mapsto w_n$ ],  
[ $v_1 \mapsto w_1, \dots, v_n \mapsto w_n, \dots$ ],  
where  $n \geq 0$ ,  $v_i : \text{type\_expr}_1$ ,  $w_i : \text{type\_expr}_2$   
May be infinite and may be non-deterministic when applied to  
elements in the domain

**NB** The original RSL book only has  $\xrightarrow{m}$ , but with the meaning of  $\xrightarrow{\sim m}$ .  
Finite maps were introduced and the symbols changed in the method  
book.

## Examples

**Nat**  $\xrightarrow{m}$  **Bool**

[ ]  
[0  $\mapsto$  true]  
[0  $\mapsto$  true, 1  $\mapsto$  true]

**Nat**  $\xrightarrow{\sim m}$  **Bool**

[ ]  
[0  $\mapsto$  true]  
[0  $\mapsto$  true, 1  $\mapsto$  true]  
[0  $\mapsto$  true, 0  $\mapsto$  false]  
[0  $\mapsto$  true, 0  $\mapsto$  false, 1  $\mapsto$  true]

## Map Value Expressions

Enumerated:  $[expr_1 \mapsto expr_1', \dots, expr_n \mapsto expr_n']$

$[3 \mapsto \text{true}, 5 \mapsto \text{false}]$

$["\text{Klaus}" \mapsto 7, "\text{John}" \mapsto 2, "\text{Mary}" \mapsto 7]$

Comprehended:  $[expr_1 \mapsto expr_2 \mid \text{typing}_1, \dots, \text{typing}_n \bullet \text{logical-expr}_3]$

$[n \mapsto 2*n \mid n : \text{Nat} \bullet n \leq 2] = [0 \mapsto 0, 1 \mapsto 2, 2 \mapsto 4]$

$[n \mapsto 2*n \mid n : \text{Nat}] = [0 \mapsto 0, 1 \mapsto 2, 2 \mapsto 4, \dots]$

## Map Application

Basic form:

$map\text{-}expr(expr_1)$

Examples:

$["\text{Klaus}" \mapsto 7, "\text{John}" \mapsto 2, "\text{Mary}" \mapsto 7]( "\text{John}" ) = 2$

$[3 \mapsto \text{true}, 3 \mapsto \text{false}](3) = \text{true} \ \square \ \text{false}$

Application is always to values in the domain; otherwise the result is non-terminating (in fact **swap**, a kind of deadlock).

## Associated Built-in Operators

$\text{dom} : (T_1 \xrightarrow{\sim m} T_2) \rightarrow T_1\text{-iniset}$      $\text{dom} [3 \mapsto \text{true}, 5 \mapsto \text{false}] = \{3, 5\}$   
 $\text{dom} [3 \mapsto \text{true}, 5 \mapsto \text{false}, 5 \mapsto \text{true}] = \{3, 5\}$

$\text{rng} : (T_1 \xrightarrow{\sim m} T_2) \rightarrow T_2\text{-iniset}$      $\text{rng} [3 \mapsto \text{false}, 5 \mapsto \text{false}] = \{\text{false}\}$   
 $\text{rng} [3 \mapsto \text{false}, 5 \mapsto \text{false}, 5 \mapsto \text{true}] = \{\text{false}, \text{true}\}$

$\dagger : (T_1 \xrightarrow{\sim m} T_2) \times (T_1 \xrightarrow{\sim m} T_2) \rightarrow (T_1 \xrightarrow{\sim m} T_2)$   
 $[3 \mapsto \text{true}, 5 \mapsto \text{false}] \dagger [5 \mapsto \text{true}] = [3 \mapsto \text{true}, 5 \mapsto \text{true}]$

$\cup : (T_1 \xrightarrow{\sim m} T_2) \times (T_1 \xrightarrow{\sim m} T_2) \rightarrow (T_1 \xrightarrow{\sim m} T_2)$   
 $[3 \mapsto \text{true}, 5 \mapsto \text{false}] \cup [5 \mapsto \text{true}] = [3 \mapsto \text{true}, 5 \mapsto \text{false}, 5 \mapsto \text{true}]$

$\setminus : (T_1 \xrightarrow{\sim m} T_2) \times T_1\text{-iniset} \rightarrow (T_1 \xrightarrow{\sim m} T_2)$      $m \setminus s = [d \mapsto m(d) \mid d : T_1 \bullet d \in \text{dom } m \wedge d \notin s]$   
 $[3 \mapsto \text{true}, 5 \mapsto \text{false}] \setminus \{5, 7\} = [3 \mapsto \text{true}]$

$/ : (T_1 \xrightarrow{\sim m} T_2) \times T_1\text{-iniset} \rightarrow (T_1 \xrightarrow{\sim m} T_2)$      $m / s = [d \mapsto m(d) \mid d : T_1 \bullet d \in \text{dom } m \wedge d \in s]$   
 $[3 \mapsto \text{true}, 5 \mapsto \text{false}] / \{5, 7\} = [5 \mapsto \text{false}]$

$\circ : (T_2 \xrightarrow{\sim m} T_3) \times (T_1 \xrightarrow{\sim m} T_2) \rightarrow (T_1 \xrightarrow{\sim m} T_3)$      $m_1 \circ m_2 = [x \mapsto m_1(m_2(x)) \mid x : T_1 \bullet x \in \text{dom } m_2 \wedge m_2(x) \in \text{dom } m_1]$   
 $[3 \mapsto \text{true}, 5 \mapsto \text{false}] \circ [ "\text{Klaus}" \mapsto 3, "\text{John}" \mapsto 7 ] = [ "\text{Klaus}" \mapsto \text{true} ]$

$\in : T_1 \times (T_1 \xrightarrow{\sim m} T_2) \rightarrow \text{Bool}$      $3 \in [3 \mapsto \text{true}] = \text{true}$

DATABASE =

**class**

**type**

Database = Key  $\overset{m}{\rightarrow}$  Data,

Key, Data

**value**

empty : Database = [ ],

insert : Key  $\times$  Data  $\times$  Database  $\rightarrow$  Database

insert(k,d,db)  $\equiv$  db  $\uparrow$  [k  $\mapsto$  d],

remove : Key  $\times$  Database  $\rightarrow$  Database

remove(k,db)  $\equiv$  db  $\setminus$  {k},

defined : Key  $\times$  Database  $\rightarrow$  **Bool**

defined(k,db)  $\equiv$  k  $\in$  db,

lookup : Key  $\times$  Database  $\overset{\sim}{\rightarrow}$  Data

lookup(k,db)  $\equiv$  db(k)

**pre** defined(k,db)

**end**

## RAISE logic

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Computing involves *partial* functions

- division
- head of a list
- loops

So we need a logic that can deal with expressions that may not be well defined.

By *well defined* we mean has (or evaluates to) a value.

## Expressions and values

An expression may or may not evaluate to a value:

| Expression  | Value       |
|---|-------------|
| <b>true</b>   | <b>true</b> |
| 1 + 0   | 1           |
| 1 / 0   | ?           |
| factorial(3)  | 6           |
| factorial(-1)   | ?           |
| factorial(x)  | ?           |
| <b>if</b> x > 0 <b>then</b> factorial(x) <b>else</b> 0 <b>end</b> | ✓           |
| <b>while true do skip end</b>                                     | ×           |

## chaos

Used to represent undefinedness

**while true do skip end**  $\equiv$  **chaos**

$/ : \mathbf{Real} \times \mathbf{Real} \rightarrow \mathbf{Real}$

1.0/0.0 is under-specified

1.0/0.0 might evaluate to **chaos**

$f : \mathbf{Real} \rightarrow \mathbf{Real}$

$f(x) \equiv$  **if** x  $\neq$  0.0 **then** 1.0/x **else** 0.0 **end**

## If expressions

Example:

**if**  $x > 0$  **then** factorial(x) **else** 0 **end**

More general form:

**if** *logical*-expr **then** expr<sub>1</sub> **else** expr<sub>2</sub> **end**

Properties:

**if true then** expr<sub>1</sub> **else** expr<sub>2</sub> **end**  $\equiv$  expr<sub>1</sub>

**if false then** expr<sub>1</sub> **else** expr<sub>2</sub> **end**  $\equiv$  expr<sub>2</sub>

**if chaos then** expr<sub>1</sub> **else** expr<sub>2</sub> **end**  $\equiv$  chaos

Non-strictness:

**if true then** expr<sub>1</sub> **else** chaos **end**  $\equiv$  expr<sub>1</sub>

**if false then** chaos **else** expr<sub>2</sub> **end**  $\equiv$  expr<sub>2</sub>

## Connectives

Definitions:

$\sim e \equiv$  **if** e **then** false **else** true **end**

$e1 \wedge e2 \equiv$  **if** e1 **then** e2 **else** false **end**

$e1 \vee e2 \equiv$  **if** e1 **then** true **else** e2 **end**

$e1 \Rightarrow e2 \equiv$  **if** e1 **then** e2 **else** true **end**

gives conditional logic

## Truth tables

| $\wedge$ | true  | false | chaos |
|----------|-------|-------|-------|
| true     | true  | false | chaos |
| false    | false | false | false |
| chaos    | chaos | chaos | chaos |

| $\vee$ | true  | false | chaos |
|--------|-------|-------|-------|
| true   | true  | true  | true  |
| false  | true  | false | chaos |
| chaos  | chaos | chaos | chaos |

| $\Rightarrow$ | true  | false | chaos |
|---------------|-------|-------|-------|
| true          | true  | false | chaos |
| false         | true  | true  | true  |
| chaos         | chaos | chaos | chaos |

## Quantified expressions

Examples:

$$\forall x : \mathbf{Nat} \bullet (x = 0) \vee (x > 0)$$

$$\exists x : \mathbf{Int} \bullet x = 7$$

$$\exists! x : \mathbf{Int} \bullet (x \geq 0) \wedge (x \leq 0)$$

$$\forall x : \mathbf{Nat} \bullet x = -7$$

$$\forall x, y : \mathbf{Nat} \bullet (\exists! z : \mathbf{Nat} \bullet x+y = z)$$

General form:

$$\text{quantifier } \text{typing}_1, \dots, \text{typing}_n \bullet \text{logical-expr}$$

Note:

$$e1 \wedge e2 \equiv e2 \wedge e1$$

$$e1 \vee e2 \equiv e2 \vee e1$$

are not tautologies

## $\equiv$ versus $=$

Assume  $e_1$  and  $e_2$  are defined, deterministic, without effects and without communication.

Assume  $e_i$  evaluates to  $v_i$ ,  $v_1 \neq v_2$ .

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| $\equiv$     | $e_1$        | $e_2$        | <b>chaos</b> |
| $e_1$        | <b>true</b>  | <b>false</b> | <b>false</b> |
| $e_2$        | <b>false</b> | <b>true</b>  | <b>false</b> |
| <b>chaos</b> | <b>false</b> | <b>false</b> | <b>true</b>  |

|              |              |              |              |
|--------------|--------------|--------------|--------------|
| $=$          | $e_1$        | $e_2$        | <b>chaos</b> |
| $e_1$        | <b>true</b>  | <b>false</b> | <b>chaos</b> |
| $e_2$        | <b>false</b> | <b>true</b>  | <b>chaos</b> |
| <b>chaos</b> | <b>chaos</b> | <b>chaos</b> | <b>chaos</b> |

All quantification is over values in the types stated, i.e. not over **chaos**.



## Use of “ $\equiv$ true”

$=$  and  $\equiv$  differ in terms of :

- ‘ $\equiv$ ’ is two valued — the result is never **chaos**
- ‘ $=$ ’ is strict, ‘ $\equiv$ ’ is not
- ‘ $\equiv$ ’ is reflexive, ‘ $=$ ’ is not

Note that “ $p \equiv \text{true}$ ” is **true** if  $p$  is, **false** otherwise, and so is always defined.

“ $\equiv \text{true}$ ” is implicitly included in some logical expressions in RSL to ensure definedness:

- axioms
- predicates in quantified expressions
- predicates in comprehended expressions
- pre and post conditions

## Conditional logic: conclusions

- **Pro**
  - Deals with undefinedness
  - Logical connectives are executable
- **Con**
  - Some classical laws require definedness:
    - \* “excluded middle”
    - \* commutativity of  $\wedge$  and  $\vee$

## Total and partial functions

total functions:

$$\text{type\_expr}_1 \rightarrow \text{type\_expr}_2$$

partial functions:

$$\text{type\_expr}_1 \overset{\sim}{\rightarrow} \text{type\_expr}_2$$

$$\forall f_{tot} : T_1 \rightarrow T_2, f_{par} : T_1 \overset{\sim}{\rightarrow} T_2, x : T_1 \cdot$$

|              |                                |               |
|--------------|--------------------------------|---------------|
|              | defined<br>(not <b>chaos</b> ) | deterministic |
| $f_{tot}(x)$ | yes                            | yes           |
| $f_{par}(x)$ | might be                       | might be      |

$$\exists! y : T_2 \cdot f_{tot}(x) \equiv y$$

## Proof rules for RAISE

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## Proof rules: purpose

- Provide formation rules to determine if a specification is well-formed
- Provide rules for reasoning:
  - is a predicate true?
  - are two terms equivalent?
  - is one specification a refinement of another?

## Axiomatic and denotational semantics

The proof rules provide an **axiomatic semantics**.

There is also a denotational semantics. Why?

- provides a model for the axiomatic semantics
- hence shows the axiomatic semantics is consistent

## Proof rules for definition: example

context  $\vdash$  value\_expr :  $\preceq$  opt\_access\_desc\_string **Bool**  
context  $\vdash$  **read-only-convergent** value\_expr

---

context  $\vdash$   
**if** value\_expr **then** value\_expr **else true end**  $\simeq$   
**true**

## Proof rule structure

$$\frac{\text{premise}_1 \dots \text{premise}_n}{\text{conclusion}}$$

meaning the conclusion is true when all the premises are.

Premises are well-formedness conditions and applicability conditions.

Conclusions are commonly equivalences, but also include typing rules and refinement relations.

## Proof rules for proof

- Aim is doing proof
- and doing so automatically as far as possible.
- Need derived rules as well as basic ones, where basic rules correspond to the axiomatic semantics rules.
- Tools can handle well-formedness and contexts: so make these implicit.

Original proof tool had about 300 basic rules, 1700+ derived ones.

## Proof rules for proof: example

```
[ if_annihilation1 ]  
if eb then eb else true end  $\simeq$  true  
when convergent(eb)  $\wedge$  readonly(eb)
```

## Proof rule structure

- Contexts and well-formedness premises have gone
- Premises introduced by **when**
- Use of *special functions* built into prover (and often automatically dischargeable)
- Conventions for term variable names, e.g.
  - e: value expression
  - b: Bool type
  - i: Int type
  - s: set type
  - e, e', e'' etc. have same type
  - e, e1, e2 etc. may have different types

## Soundness

Which of these rules are sound?

```
[subset_difference]
es ⊆ es' \ es'' ≈ true
  when convergent(es) ∧ readonly(es) ∧
    convergent(es') ∧ readonly(es') ∧
    convergent(es'') ∧ readonly(es'') ∧ es ⊆ es' ∧ es ∩ es'' = {}

[proper_subset_difference]
es ⊂ es' \ es'' ≈ true
  when convergent(es) ∧ readonly(es) ∧
    convergent(es') ∧ readonly(es') ∧
    convergent(es'') ∧ readonly(es'') ∧ es ⊂ es' ∧ es ∩ es'' = {}
```

## More soundness tests

```
[subset_expansion1]
es ⊆ es' ≈ ~ (es ⊃ es')

[proper_subset_expansion1]
es ⊂ es' ≈ ~ (es ⊇ es')
```

## A problem

How do you find all the errors on 2000+ rules?

## One answer

Use another theorem prover, assume faults are independent, and prove your proof rules.

Using the PVS translator, found 6 errors affecting 11 rules in 1000+.

### Another problem

How do you know the built-in procedures in your proof tool are sound?

### One answer

1. Use a proof tool that has built-in procedures and can output the proof in terms of basic proof rules.
2. Rerun the proof in another prover with no procedures and only basic proof rules.

This only helps with individual proofs: correspond to test cases for the proof tool.

But could be used on a critical project.

## Imperative RSL

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## Imperative Specification: Example

```
COUNTER =  
class  
  variable  
    counter : Nat := 0  
  value  
    increase : Unit → write counter Nat  
    increase() ≡ counter := counter + 1 ; counter  
end
```

## Imperative Expressions

No syntactic distinction between

- statements and
- expressions

Imperative expressions:

- assignments ( $id := value\_expr$ )
- sequencing ( $unit\_value\_expr_1 ; value\_expr_2$ )
- iterative expressions (while, until, for)
- if expressions
- ...

## Meanings of expressions

In general expressions may have both

- **effects** and
- **values**

**Effects** are changes to variables and input or output on channels.

For expressions to be **equivalent** ( $\equiv$ ) they must have the same potential effects as well as the same values.

For expressions to be **equal** ( $=$ ) they must have the same values.

## Evaluation Order

Evaluation order is critical when we may have effects.

Evaluation in RSL is **left-to-right**.

For example, suppose we have a **variable**  $x : \text{Int}$ :

$$\langle x := 1 ; x , x := 2 ; x \rangle \equiv x := 2 ; \langle 1, 2 \rangle$$

$$\langle x := 2 ; x , x := 1 ; x \rangle \equiv x := 1 ; \langle 2, 1 \rangle$$

$$x + (x := x + 1 ; x) \equiv x := x + 1 ; 2 * x - 1$$

$$(x := x + 1 ; x) + x \equiv x := x + 1 ; 2 * x$$

## Equivalence versus Equality

$=$  and  $\equiv$  differ in terms of

- undefinedness (**chaos**)
- non-determinism
- effects (variables and communication)

otherwise they are the same.

For example, we can say

$\text{factorial}(3) = 6$

or

$\text{factorial}(3) \equiv 6$

They are both true.

## When equivalence and equality differ

Assume the variable  $x$  currently holds the value 0.

| Expression  | Evaluation                           |
|---|--------------------------------------|
| $1 \parallel 2 = 1 \parallel 2$                         | <b>true</b> $\parallel$ <b>false</b> |
| $1 \parallel 2 \equiv 1 \parallel 2$                    | <b>true</b>                          |
| <b>while true do skip end = chaos</b>                   | <b>chaos</b>                         |
| <b>while true do skip end <math>\equiv</math> chaos</b> | <b>true</b>                          |
| $((x := x + 1 ; 1) = (x := x + 1 ; x))$                 | $x := 2 ;$ <b>false</b>              |
| $((x := x + 1 ; 1) \equiv (x := x + 1 ; x))$            | <b>true</b>                          |

## Equivalence versus Equality

- $\equiv$  and  $=$  are the same if the arguments are convergent and pure.
- $\equiv$  is always defined.
- $\equiv$  compares effects as well as results;  $=$  only compares results
- $\equiv$  has hypothetical evaluation;  $=$  has left-to-right evaluation.
- $\equiv$  gives no effects;  $=$  may give effects.

## Applicative to imperative transformation

- Remove definition of type of interest.
- Add variable(s) to model concrete type of interest.
- Remove type of interest from function signatures; fill holes with **Unit**.
- Give type "**Unit**  $\rightarrow$  **write** ... **Unit**" to each constant "c" of type of interest, and replace "=" by "c()  $\equiv$ ".
- Insert write accesses to generators.
- Insert read accesses to observers.
- Remove formal parameters representing type of interest.
- Replace occurrences of type of interest parameters with references to variable(s).

- For generators, insert assignments.

This is for "leaf" modules in a hierarchy. Non-leaf modules involves a similar but simpler transformation.

## Imperative example

```
I_DATABASE =  
class  
  type Key, Data  
  variable database : Key  $\rightsquigarrow$  Data  
  value  
    empty : Unit  $\rightarrow$  write database Unit  
    empty()  $\equiv$  database := [],  
  
    insert : Key  $\times$  Data  $\rightarrow$  write database Unit  
    insert(k,d)  $\equiv$  database := database  $\uparrow$  [k  $\mapsto$  d],  
  
    remove : Key  $\rightarrow$  write database Unit  
    remove(k)  $\equiv$  database := database  $\setminus$  {k},
```

```
defined : Key  $\rightarrow$  read database Bool  
defined(k)  $\equiv$  k  $\in$  dom database,
```

```
lookup : Key  $\rightsquigarrow$  read database Data  
lookup(k)  $\equiv$  database(k)
```

```
pre defined(k)  
end
```



## While expressions

```
FRACTION_SUM = class
  variable
    counter : Nat, result : Real
  value
    fraction_sum : Nat  $\rightsquigarrow$  write counter, result Unit
    fraction_sum(n)  $\equiv$ 
      counter := n ; result := 0.0 ;
      while counter > 0 do
        result := result + 1.0/(real counter) ;
        counter := counter - 1
      end
    pre n > 0
  end
```

$$1 + 1/2 + \dots + 1/n$$

Is the precondition of fraction\_sum necessary?

## Until Expressions

```
FRACTION_SUM = class
  variable
    counter : Nat, result : Real
  value
    fraction_sum : Nat  $\rightsquigarrow$  write counter, result Unit
    fraction_sum(n)  $\equiv$ 
      counter := n ; result := 0.0 ;
      do
        result := result + 1.0/(real counter) ;
        counter := counter - 1
      until counter = 0 end
    pre n > 0
  end
```

## For Expressions

```
FRACTION_SUM =
  class
    variable
      result : Real
    value
      fraction_sum : Nat  $\rightsquigarrow$  write result Unit
      fraction_sum(n)  $\equiv$ 
        result := 0.0 ;
        for i in <1 .. n> do
          result := result + 1.0/(real i)
        end
      pre n > 0
    end
  end
```

## Concurrency

### Concurrent RSL

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Concurrency is necessary in particular for describing distributed systems.

Concurrent systems in general may communicate through

- shared variables, or
- message passing

RSL uses message passing.

Message passing is more abstract: shared variables may be modelled using message passing.

## Composition of Expressions

Composition:

- sequential:

$\text{value\_expr}_1 ; \text{value\_expr}_2$

- concurrent:

$\text{value\_expr}_1 \parallel \text{value\_expr}_2$

1. has type **Unit**
2.  $\text{value\_expr}_1$  and  $\text{value\_expr}_2$  must have type **Unit**
3.  $\text{value\_expr}_1$  and  $\text{value\_expr}_2$  recommended to be assignment-disjoint

## Communication Expressions

**channel** id : type\_expr

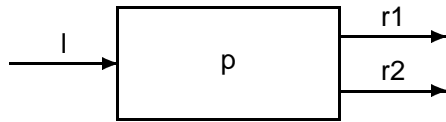
Communication expressions:

- input expressions: id ?
- output expressions: id ! value\_expr

Input expressions have the same type as the channel.

Output expressions have type **Unit**.

### Example

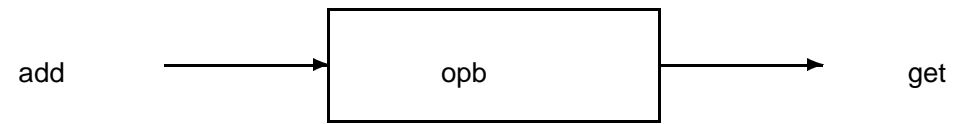


```

channel
  l, r1, r2: Int

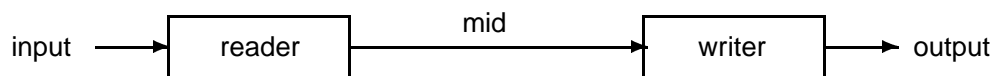
value
  p : Unit → in l out r1, r2 Unit
  p() ≡
    let e = l? in (r1!e || r2!e) end; p()
  
```

### Another example



```

ONE_PLACE_BUFFER =
class
  type Elem
  channel add, get : Elem
  value
    opb : Unit → in add out get Unit
    opb() ≡ let v = add? in get!v end ; opb()
  end
  
```



```

READER_WRITER =
class
  type Elem
  channel input, output, mid : Elem
  value
    reader : Unit → in input out mid Unit
    reader() ≡
      let v = input? in mid ! v end ; reader(),
    writer : Unit → in mid out output Unit
    writer() ≡
      let v = mid? in output ! v end ; writer()
  end
  
```

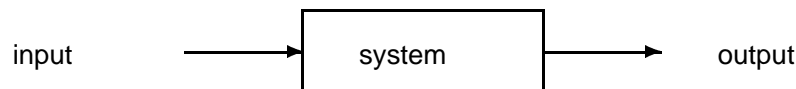
```

SYSTEM = extend READER_WRITER with
class
  value
    system : Unit →
      in input, mid out output, mid Unit
    system() ≡ reader() || writer()
  end
  
```

```

system() ≡
  let v = input? in mid ! v end ; reader()
  ||
  let v = mid? in output ! v end ; writer()
  
```

We should make the channel *mid* unavailable to any other processes.



```

SYSTEM = class
  type Elem
  channel input, output : Elem
  value
    system : Unit → in input out output Unit
    system() ≡
      local
        channel mid : Elem
        value
          reader : Unit → in input out mid Unit
          reader() ≡ let v = input? in mid ! v end ; reader(),
          writer : Unit → in mid out output Unit
          writer() ≡ let v = mid? in output ! v end ; writer()
        in reader() || writer() end
  end

```

### External choice

The value expression

```
v:=c1? [] c2!e
```

will:

- input from *c1* if a value expression is willing to output to *c1* but no value expression is willing to input from *c2*;
- output to *c2* if a value expression is willing to input from *c2* but no value expression is willing to output to *c1*;
- either input from *c1* or output to *c2* if a value expression is willing to output to *c1* and a value expression is willing to input from *c2*;
- deadlock if no value expression is ever willing to output to *c1* and no value expression is ever willing to input from *c2*.

### Internal choice

The value expression

```
v:=c1? [] c2!e
```

will:

- either deadlock or input from *c1* if a value expression is willing to output to *c1* but no value expression is willing to input from *c2*;
- either deadlock or output to *c2* if a value expression is willing to input from *c2* but no value expression is willing to output to *c1*;
- either input from *c1* or output to *c2* if a value expression is willing to output to *c1* and a value expression is willing to input from *c2*;
- deadlock if no value expression is ever willing to output to *c1* and no value expression is ever willing to input from *c2*.

```

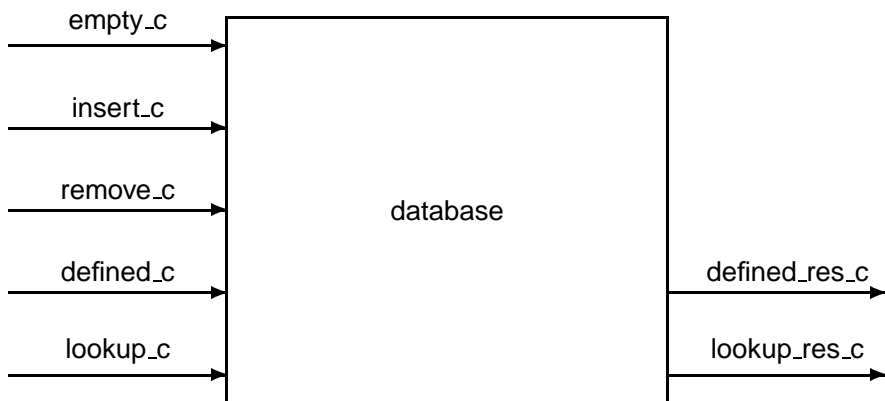
channel
  empty : Unit, add, get : Elem
value
  mpb : Unit → in empty, add out get Unit
  mpb() ≡
    local
      variable buffer : Elem* := ⟨⟩
    in
      while true do
        empty? ; buffer := ⟨⟩
        []
        let v = add? in buffer := buffer ^ ⟨v⟩ end
        []
        if buffer ≠ ⟨⟩ then get ! hd buffer ; buffer := tl buffer
        else stop end
      end
    end
  end
end

```

### Typical Development

|          | Applicative | Imperative | Concurrent |
|----------|-------------|------------|------------|
| Abstract |             |            |            |
| Concrete |             |            |            |

↓ Refinement      - - - - - Transformation



### Imperative to concurrent transformation

- Insert an object instantiating the imperative sequential module, and hide it.
- Define channels for each observer and generator; at least one channel for each. Hide them.
- Define a “server” process:
  - type “Unit → in ... out ... write l.any Unit”
  - body is a while true do loop
  - loop body is an external choice between clauses, one clause for each observer and each generator
  - each clause inputs parameters (if any); calls corresponding function l.f; outputs result (if any). Must do at least one communication.

Hide it.

- Define an “init” process with the same type as the server that initialises the imperative object and calls the server.
- Define “interface functions” mirroring clauses in server. These *have no accesses to the imperative object*.

This is for “leaf” modules in a hierarchy. Non-leaf modules are similar but easier.

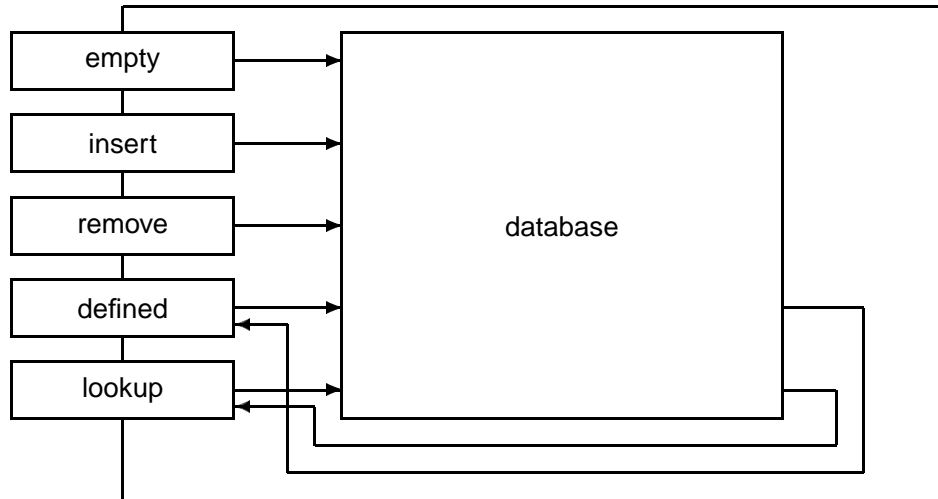
```
C_DATABASE = hide I, database in
class
  object I : I.DATABASE
  type
    Key = I.Key,
    Data = I.Data,
    Result == not_found | res(Data)
  channel
    empty_c : Unit,
    insert_c : Key × Data,
    remove_c, defined_c, lookup_c : Key,
    defined_res_c : Bool,
    lookup_res_c : Result
```

**value**

```
init : Unit → in empty_c, insert_c, remove_c, defined_c, lookup_c
      out defined_res_c, lookup_res_c write I.any Unit
init() ≡ I.empty() ; database(),
```

```
database : Unit → in ... out ... write I.any Unit
database() ≡
  while true do
    empty_c? ; I.empty()
    []
    let (k,d) = insert_c? in I.insert(k,d) end
    []
    let k = remove_c? in I.remove(k) end
    []
    let k = defined_c? in defined_res_c ! I.defined(k)
  end
  []
  let k = lookup_c? in
    if I.defined(k) then lookup_res_c ! res(I.lookup(k))
    else lookup_res_c ! not_found end end end end
```

## Encapsulation with Interface Functions



```

INTERFACED_DATABASE =
  hide empty_c, insert_c, remove_c, defined_c, lookup_c,
        defined_res_c, lookup_res_c in
  extend C.DATABASE with
  class
    value
      empty : Unit → out any Unit
      empty() ≡ empty_c ! (),

      insert : Key × Data → out any Unit
      insert(k,d) ≡ insert_c ! (k,d),

      remove : Key → out any Unit
      remove(k) ≡ remove_c ! k,
  
```

```

defined : Key → in any out any Bool
defined(k) ≡ defined_c ! k ; defined_res_c?,
  
```

```

lookup : Key → in any out any Result
lookup(k) ≡ lookup_c ! k ; lookup_res_c?
  
```

end

## Modularity in RSL

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An RSL specification consists of

- module definitions

A module contains definitions of

- types
- values
- variables
- channels
- modules
- axioms

## Modularity

Modules are the building blocks.

Purposes:

- Readability
- Separate development
- Reuse

## Schemes and Objects

Modules are either schemes or objects.

A scheme denotes a class of models

**scheme** id = class\_expr

An object denotes a single model

**object** id : class\_expr



## Class Expressions

- basic
- with
- extending
- renaming
- hiding
- instantiation

## With class expression

General form:

**with** *element-object\_expr-list* **in** *class\_expr*

**with** *X* **in** *class\_expr*

means that a name *n* in *class\_expr* can mean either *n* or *X.n*. This means, providing there is no confusion, that qualifications like *X*. can be omitted.

## Extension

General form:

**extend** *class\_expr<sub>1</sub>* **with** *class\_expr<sub>2</sub>*

appends the second class to the first.

*class\_expr<sub>1</sub>* and *class\_expr<sub>2</sub>* must be compatible

## Renaming

General form:

**use**  
*id<sub>new<sub>1</sub></sub>* **for** *id<sub>old<sub>1</sub></sub>*, ... , *id<sub>new<sub>n</sub></sub>* **for** *id<sub>old<sub>n</sub></sub>*  
**in** *class\_expr*

For example

```
scheme BUFFER =  
  use  
    add for enq, get for deq, Buffer for Queue  
  in QUEUE
```

## Hiding

General form:

```
hide id1,...,idn in class_expr
```

Hidden entities

1. are not visible outside
2. need not be implemented

Typically use:

1. prevention of unintended access to variables and/or channels
2. hiding of auxiliary functions

## Objects

```
scheme BUFFER =  
  class  
    variable buff : Int*  
    value  
      is_empty : Unit → read buff Bool  
      ...  
  end  
  
  object  
    B1 : BUFFER,  
    B2 : BUFFER
```

B1 and B2 are distinct, global objects and we can use them ...

## Using objects

```
scheme SYS =  
  class  
    value  
      one_is_empty : Unit → read B1.buff B2.buff Bool  
      one_is_empty() ≡ B1.is_empty() ∨ B2.is_empty()  
  end
```

B1.buff and B2.buff are distinct

## Module Nesting

```
scheme  
  SYS =  
    class  
      object  
        B1 : BUFFER,  
        B2 : BUFFER  
      value  
        one_is_empty : Unit → read B1.buff B2.buff Bool  
        one_is_empty() ≡ B1.is_empty() ∨ B2.is_empty()  
    end
```

B1 and B2 are distinct, embedded objects.

## Building hierarchies

Suppose we have a system that needs a database component.

There are several ways we can construct the specification:

- merging the system and database definitions in one class
- extending the database class with the system class
- making a hierarchy with a database object

## Merging the definitions in one class

```
scheme SYSTEM =  
  class  
    /* database */  
    :  
    /* system */  
    :  
  end
```

- Hard to read
- Database cannot be reused
- Hard to make database private to system
- Problem of name clashes between two parts

## Extending the database

```
scheme DATABASE = ...  
  
scheme SYSTEM =  
  extend DATABASE with ...
```

- Easier to read
- Database can be reused
- Hard to make database private to system
- Problem of name clashes between two parts

## Making a hierarchy with a database object

```
scheme DATABASE = ...  
  
scheme SYSTEM =  
  class  
    object DB : DATABASE  
    :  
  end
```

- Easier to read
- Database can be reused
- Easy to make database private to system:

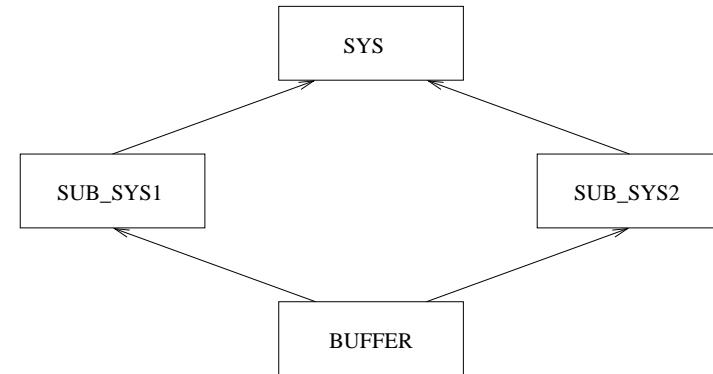
```

scheme SYSTEM =
  hide DB in
  class
    object DB : DATABASE
    :
  end

```

- No problem of name clashes between two parts

## Sharing



```

scheme BUFFER = ...

scheme SUB_SYS1 = class object B : BUFFER ... end
scheme SUB_SYS2 = class object B : BUFFER ... end

scheme SYS =
  class
    object
      O1 : SUB_SYS1,
      O2 : SUB_SYS2
    :
  end

```

We get two buffer variables (O1.B.buff and O2.B.buff)

## Sharing using global objects

```

object B : BUFFER

scheme
  SUB_SYS1 = class ... B.buff ... end,
  SUB_SYS2 = class ... B.buff ... end,

  SYS = class
    object
      O1 : SUB_SYS1,
      O2 : SUB_SYS2
    :
  end

```

We get only one buffer: B.buff

## Sharing using parameterization

```
scheme BUFFER = ...
scheme SUB_SYS1(B: BUFFER) = ...
scheme SUB_SYS2(B: BUFFER) = ...

scheme SYS =
  class
    object
      B : BUFFER,
      O1 : SUB_SYS1(B),
      O2 : SUB_SYS2(B)
      :
    end
```

## Parameterization - Example

```
scheme BUFFER =
  class
    type Elem
    variable buff : Elem*
    value
      empty : Unit → write buff Unit
      empty() ≡ buff := ⟨⟩,

      add : Elem → write buff Unit
      add(e) ≡ buff := buff ^ ⟨e⟩
    end
```

is better expressed using parameterization:

```
scheme ELEM = class type Elem end

scheme BUFFER(E : ELEM) =
  class
    variable buff : E.Elem*
    value
      empty : Unit → write buff Unit
      empty() ≡ buff := ⟨⟩,

      add : E.Elem → write buff Unit
      add(e) ≡ buff := buff ^ ⟨e⟩
    end
```

## Instantiation - Example

```
object
  INTEGER :
    class
      type Elem = Int
    end,

  INTEGER_BUFFER : BUFFER(INTEGER)
```

If we expand BUFFER(INTEGER):

INTEGER\_BUFFER :

**class**

**variable** buff : INTEGER.Elem\*

**value**

empty : **Unit** → **write** buff **Unit**

empty() ≡ buff := ⟨⟩,

add : INTEGER.Elem → **write** buff **Unit**

add(e) ≡ buff := buff ^ ⟨e⟩

**end**

## Actual versus Formal Parameters

**scheme** S(X : FC)

**object** A : AC,

... S(A) ...

Context condition: AC must statically implement FC

## RAISE Method

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- *The licensed material is provided “as is” without warranty of any kind.*
- *The Vendor disclaims ... conformance between the software and ... manuals*
- *The entire risk ... is with the Licensee*
- *... in no event will the Vendor be liable for any damages ...*
- *The Licensee shall ... hold harmless the Vendor against all claims ...*

Claims of competence, perhaps?

## Software Crisis

- for every six new large-scale software systems that are put in operation, two others are cancelled.
- the average software development project overshoots its schedule by half
- some three quarters of all large systems do not function as intended or are not used at all.

“**Software Hell** – Bugs. Viruses. Complexity.  
Is there any way out of this mess?” (Business Week, 1999)

## Denver Airport Baggage-Handling, 1994

- Twice the size of Manhattan, 10 times the breadth of Heathrow, three jets can land simultaneously in bad weather.
- The subterranean baggage-handling system consists of 34 km of track with 4000 independent “telecars” routing and delivering luggage between counters, gates and claim areas. It is controlled by a network of 100 computers with 5000 sensors, 400 radio receivers and 56 bar-code scanners.
- Despite his woes, the contractor says the project’s worth it: “Who would turn down a USD 193 million contract? You’d expect to have a little trouble for that kind of money.” (New York Times, 18 Mar 1994)

## Denver Airport Baggage-Handling, 1994 (cont.)

- Software did not work!
- Too little time for system testing.
- The delay of the opening of the airport was 9 months.
- They decided to build **another** baggage handling system — the conventional kind with conveyor belts — for another USD 50 million.

## Ariane 5, 1996

- On 4 June 1996 Ariane 5 rocket exploded,
- Caused by software in the inertial guidance system.
- An inertial platform from the Ariane 4 was used aboard the Ariane 5 **without proper testing**.
- When subjected to the higher accelerations produced by the Ariane 5 booster, the software (calibrated for an Ariane 4) ordered an "abrupt turn 30 seconds after liftoff".
- A **precondition** of the software was violated.

## Mars Climate Orbiter, 1999

- Sept. 1999 Mars Climate Orbiter disappeared after successfully travelling 416 million miles in 41 weeks.
- Lockheed Martin Astronautics used acceleration data in **Imperial units** (feet per second per second).
- Jet Propulsion Laboratory (JPL) did its calculations with **metric units** (metres per second per second).
- Integration testing should have been revealed this fault!
- NASA started a \$50,000 project to discover how this could have happened.

What is the acceleration due to gravity?



The stories continue

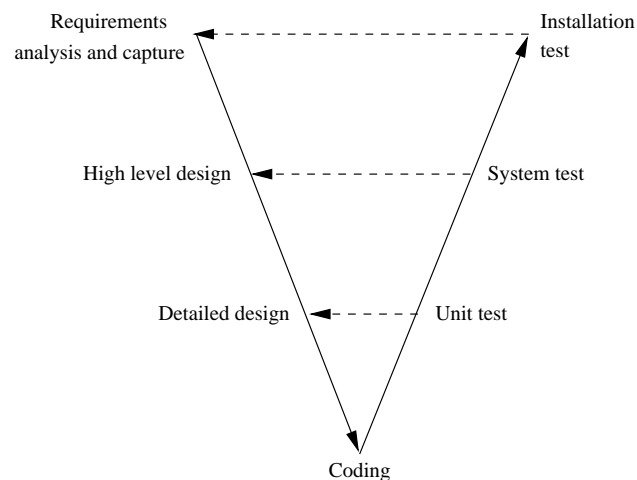
Peter Neumann's Risk Forum

<http://catless.ncl.ac.uk/Risks/>

In academia, in industry, and in the commercial world, there is a widespread belief that computing science as such has been all but completed and that, consequently, computing has matured from a theoretical topic for the scientists to a practical issue for the engineers, the managers, and the entrepreneurs. [...]

I would therefore like to posit that computing's central challenge, "How not to make a mess of it," has not been met. On the contrary, most of our systems are much more complicated than can be considered healthy, and are too messy and chaotic to be used in comfort and confidence. The average customer of the computing industry has been served so poorly that he expects his system to crash all the time, and we witness a massive worldwide distribution of bug-ridden software for which we should be deeply ashamed. (Communications of the ACM, Mar 2001)

V-diagram model of software life cycle



The V-diagram illustrates the typical re-work cycles when we discover errors by testing.

We aim to *find errors earlier*.

We concentrate on the early stages:

- *requirements analysis and capture*
- *high level design*

## Why formal methods?

To produce software that is

- more likely to be correct
- more reliable
- better documented
- more easily maintainable

## What is formality?

- a language — symbols and grammar rules for constructing terms
- (usually) rules for deciding if terms are well formed (e.g. scope, typing rules)
- a semantics — a description of what terms mean
- a logic — a set of rules for determining if predicates about terms are true

Programming languages are not formal according to this definition because they lack a logic.

## Characteristics of formal methods

- Precise notation
- Abstraction (*what* rather than *how*)
- Stepwise development (gradual commitment)
- Proof opportunities and justifications
- Structuring based on compositionality
- Guidelines for quality assurance

## Rigorous methods

Choice of level of formality. E.g.

1. No proof opportunities generated or checked
2. Proof opportunities generated and inspected but not proved
3. Proof opportunities generated and proved with some informal steps — “it follows immediately that ...”
4. Proof opportunities generated and proved formally

All formal methods are in fact rigorous. But only a method with a formal basis can be rigorous, because it must always be possible to say “I am not sure if it does follow. Please prove it.”

Current state of the art is the first three levels.

## Implementation relation

- new signature includes the old one (statically decidable)
- old properties preserved by the new one ( $\Rightarrow$  implementation conditions)

## Example 1

```
scheme S0 =  
  class  
    value x : Int  
    axiom x  $\geq$  0  
  end
```

```
scheme S2 =  
  class  
    value  
      x : Int = 2  
      y : Int = 0  
  end
```

```
scheme S1 =  
  class  
    value  
      x : Int = 2  
  end
```

Does S1 or S2 implement S0?

Does S2 implement S1?

## Example 2

```
scheme S0 =  
  hide z in class  
    value x, y, z : Int  
    axiom x > z  $\wedge$  z > y  
  end
```

```
scheme S2 =  
  class  
    value  
      x : Int = 2  
      y : Int = 0  
  end
```

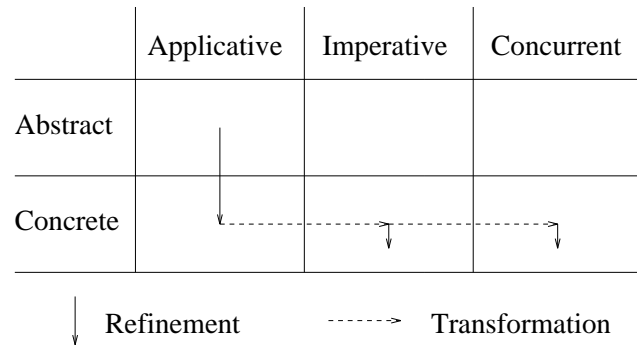
```
scheme S1 =  
  class  
    value  
      x : Int = 1  
      y : Int = 0  
  end
```

Does S1 or S2 implement S0?

## Design

- removing underspecification
  - abstract types to concrete types
  - more explicit value definitions
- changing style
  - applicative/imperative
  - sequential/concurrent
- providing more efficient algorithms

## Typical Development



## Translation

- manual translation
  - automatic translation (to SML and C++)
- of low-level RSL (e.g. concrete types and explicit value definitions)

## An example

## An example RAISE development

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A message system with possible overtaking:

- Messages can be inserted and extracted.
- There may be some delay between a message being inserted and it being available for extraction.
- The extraction order should be the same as the insertion order, except that there should be some possibility of higher priority messages “overtaking” lower priority ones.
- It is not necessary to guarantee that the next message extracted is the highest priority one in the system. This is ideal, but may not always be possible.

## TYPES

```
scheme
TYPES =
  class
    type Message

  value
    priority : Message → Nat,

    leq : Message × Message → Bool
    leq(m1, m2) ≡ priority(m1) ≤ priority(m2)
end
```

## A\_MESSAGE0

```
scheme A_MESSAGE0 =
  hide buffered, permutation, count in
  class
    type Buffer
    value
      put : T.Message × Buffer → Buffer,
      get : Buffer → T.Message × Buffer,
      can_get : Buffer → Bool,
      buffered : Buffer × T.Message* → Bool
    axiom
      [can_get_ax]
      ∀ buff : Buffer • buffered(buff, ⟨⟩) ⇒ ~ can_get(buff),
```

[buffered\_put]  
 $\forall \text{buff, buff}' : \text{Buffer}, l : \text{T.Message}^*, m : \text{T.Message} \bullet$   
 $\text{buffered}(\text{buff}, l) \wedge \text{put}(m, \text{buff}) = \text{buff}' \Rightarrow \text{buffered}(\text{buff}', l \hat{\ } \langle m \rangle),$

[buffered\_get]  
 $\forall \text{buff, buff}' : \text{Buffer}, l : \text{T.Message}^*, m1, m2 : \text{T.Message} \bullet$   
 $\text{buffered}(\text{buff}, l) \wedge \text{can\_get}(\text{buff}) \wedge \text{get}(\text{buff}) = (m1, \text{buff}') \Rightarrow$   
 $(\exists l1, l2 : \text{T.Message}^* \bullet$   
 $l = l1 \hat{\ } \langle m1 \rangle \hat{\ } l2 \wedge \text{buffered}(\text{buff}', l1 \hat{\ } l2) \wedge$   
 $(m2 \in \text{elems } l1 \Rightarrow \sim \text{T.leq}(m1, m2))),$

[no\_loss\_or\_gain]  
 $\forall \text{buff} : \text{Buffer}, l1, l2 : \text{T.Message}^* \bullet$   
 $\text{buffered}(\text{buff}, l1) \wedge \text{buffered}(\text{buff}, l2) \Rightarrow \text{permutation}(l1, l2)$

end

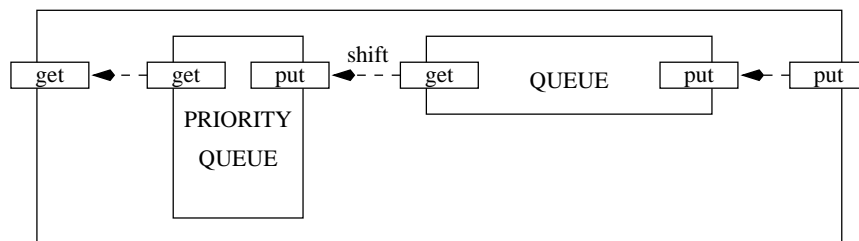
*buffered* is a relation between the abstract state *Buffer* and the list of messages input but not yet extracted.

This is naturally a relation (rather than a function) because it is many-many. Suppose *m1* and *m2* are messages, with *m2* higher priority.

Output *m2* followed by *m1* has two possible inputs.

Input *m1* followed by *m2* has two possible outputs.

## Design idea



## Parameter classes

scheme ELEM = class type Elem end

```

scheme A_QUEUE(E : ELEM) =
  class
    type Queue = E.Elem*
    value
      empty : Queue = ⟨⟩,

      put : E.Elem × Queue → Queue
      put(e, s) ≡ s ^ ⟨e⟩,

      get : Queue → E.Elem × Queue
      get(s) ≡ (hd s, tl s) pre ~ is_empty(s),

      is_empty : Queue → Bool
      is_empty(s) ≡ s = ⟨⟩
  end

```

```

scheme
  PARTIAL_ORDER(E : ELEM) =
    class
      value
        leq : E.Elem × E.Elem → Bool

      axiom
        [reflexive] ∀ a : E.Elem • leq(a, a),

        [transitive]
          ∀ a, b, c : E.Elem • leq(a, b) ∧ leq(b, c) ⇒ leq(a, c)
    end

```

```

scheme
  TOTAL_ORDER(E : ELEM) =
    extend PARTIAL_ORDER(E) with
      class
        axiom
          [linear] ∀ a, b : E.Elem • leq(a, b) ∨ leq(b, a)
      end

```

```

scheme A_PRI_QUEUE(E : ELEM, T : TOTAL_ORDER(E)) =
  hide is_ordered in
  class
    type Pri_queue = { | l : E.Elem* • is_ordered(l) | }

    value
      is_ordered : E.Elem* → Bool
      is_ordered(l) ≡
        (∀ i, j : Nat • {i, j} ⊆ inds l ∧ i < j ⇒ T.leq(l(j), l(i))),

```

```

empty : Pri_queue =  $\langle \rangle$ ,

put : E.Elem  $\times$  Pri_queue  $\rightarrow$  Pri_queue
put(e, s)  $\equiv$ 
  case s of
     $\langle \rangle \rightarrow \langle e \rangle$ ,
     $\langle h \rangle \wedge t \rightarrow$  if T.leq(e, h) then  $\langle h \rangle \wedge$  put(e, t) else  $\langle e, h \rangle \wedge t$  end
  end,

get : Pri_queue  $\xrightarrow{\sim}$  E.Elem  $\times$  Pri_queue
get(s)  $\equiv$  (hd s, tl s) pre  $\sim$  is_empty(s),

is_empty : Pri_queue  $\rightarrow$  Bool
is_empty(s)  $\equiv$  s =  $\langle \rangle$ 

end

```

## Confidence conditions

| RSL  | Confidence condition                               |
|--|--|
| value<br>empty : Pri_queue = $\langle \rangle$ | is_ordered( $\langle \rangle$ )                    |
| hd s   | s $\neq$ $\langle \rangle$                         |
| PQ.get(s)                                      | PQ.is_ordered(s) $\wedge$<br>$\sim$ PQ.is_empty(s) |

## Concrete applicative message system

```

scheme A_MESSAGE1 =
  hide PQ, Q in
  class
    object
      PQ : A_PRI_QUEUE(T{Message for Elem}, T),
      Q : A_QUEUE(T{Message for Elem})

    type Buffer = PQ.Pri_queue  $\times$  Q.Queue

    value
      put : T.Message  $\times$  Buffer  $\rightarrow$  Buffer
      put(m, (pq, q))  $\equiv$  (pq, Q.put(m, q)),

```

```

get : Buffer  $\xrightarrow{\sim}$  T.Message  $\times$  Buffer
get(pq, q)  $\equiv$ 
  let (e, pq') = PQ.get(pq) in (e, (pq', q)) end
  pre can_get(pq, q),

can_get : Buffer  $\rightarrow$  Bool
can_get(pq, q)  $\equiv$   $\sim$  PQ.is_empty(pq),

shift : Nat  $\times$  Buffer  $\rightarrow$  Buffer
shift(n, (pq, q))  $\equiv$ 
  if n = 0  $\vee$  Q.is_empty(q) then (pq, q)
  else
    let (m, q') = Q.get(q), pq' = PQ.put(m, pq) in
      shift(n - 1, (pq', q'))
    end end

end

```



## Implementation relation

Class B *implements* a class A (written  $B \preceq A$ ) if and only if

1. the signature of B includes the signature of A
2. all the *properties* of A hold in B

The signature check is static and done by tools.

## Properties of a class

These arise from

- axioms
- value definitions
- subtype conditions on values, variables and channels
- initialisations of variables
- properties of objects defined in the class

## Showing A\_MESSAGE1 $\preceq$ A\_MESSAGE0

Extend A\_MESSAGE1 with a definition of *buffered*:

```
value
buffered : Buffer × T.Message* → Bool
buffered((pq, q), l) ≡
  (∃ l1 : T.Message* • l = l1 ^ q ∧ pq = sort(l1)),

sort : T.Message* → T.Message*
sort(l) ≡
  if l = ⟨⟩ then ⟨⟩
  else
    let i = first(l) in
      ⟨l(i)⟩ ^ sort(sublist(l, 1, i - 1) ^ sublist(l, i + 1, len l))
  end
end
```

```

first : T.Message*  $\rightarrow$  Nat
first(l) as i post
  i  $\in$  inds l  $\wedge$ 
  ( $\forall j : \text{Nat} \bullet j \in \{1 \dots i - 1\} \Rightarrow \sim \text{T.leq}(l(i), l(j))$ )  $\wedge$ 
  ( $\forall j : \text{Nat} \bullet j \in \{i + 1 \dots \text{len } l\} \Rightarrow \text{T.leq}(l(j), l(i))$ )
pre l  $\neq$   $\langle \rangle$ ,

```

```

sublist : T.Message*  $\times$  Nat  $\times$  Nat  $\rightarrow$  T.Message*
sublist(l, i, j) as l1 post
  if i < 1  $\vee$  j > len l  $\vee$  i > j then l1 =  $\langle \rangle$ 
  else
    len l1 = j - i + 1  $\wedge$ 
    ( $\forall k : \text{Nat} \bullet k \in \text{inds } l1 \Rightarrow l1(k) = l(k + i - 1)$ )
  end

```

and copy definitions of *permutation* and *count* to the extension.

Can then generate 4 axioms of A\_MESSAGE0 as properties to prove of extended A\_MESSAGE1 (with definitions of *sort*, *first*, *sublist*, *permutation* and *count*).

**NB:** Extension must be *conservative*.

## Theories

RAISE allows *theories* to be stated and used in justifications.

```

[permutation_transitive]
in A_MESSAGE1_EXT  $\vdash$ 
   $\forall l1, l2, l3 : \text{T.Message}^* \bullet$ 
    permutation(l1, l2)  $\wedge$  permutation(l2, l3)  $\Rightarrow$  permutation(l1, l3),

```

```

[count_concatenation]
in A_MESSAGE1_EXT  $\vdash$ 
   $\forall m : \text{T.Message}, l1, l2 : \text{T.Message}^* \bullet$ 
    count(m, l1 ^ l2) = count(m, l1) + count(m, l2)

```

## “Internal” functions

We have added *shift* to the interface, but we expect it to become internal. What properties should it have?

*shift* is not “invisible”: can change *can\_get*, for example.

Use the relation *buffered*:

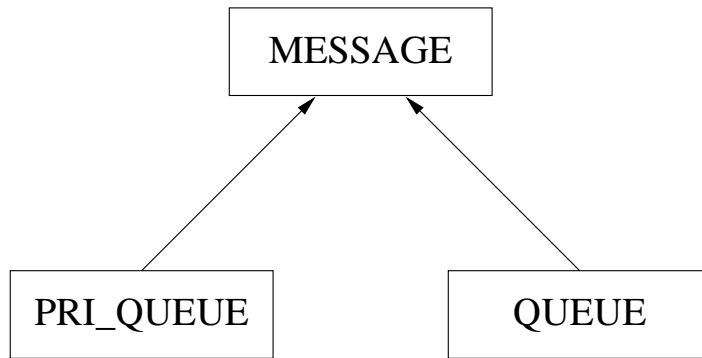
```

 $\forall \text{buff}, \text{buff}' : \text{Buffer}, n : \text{Nat} \bullet$ 
  buff' = shift(n, buff)  $\Rightarrow$ 
  ( $\exists l : \text{T.Message}^* \bullet \text{buffered}(\text{buff}, l) \wedge \text{buffered}(\text{buff}', l)$ )

```

No loss or gain of messages.

## Module dependencies



## Applicative to imperative

- when all types of interest are concrete
- module by module: preserves structure
- “leaf” modules will have imperative state
- transformation: *correct by construction*

## Imperative sequential queue

```
scheme I.QUEUE(E : ELEM) =  
  hide v, Q in class  
    object Q : A.QUEUE(E)  
  
    variable v : Q.Queue := Q.empty  
  
    value  
      empty : Unit → write any Unit  
      empty() ≡ v := Q.empty,  
  
      put : E.Elem → write any Unit  
      put(e) ≡ v := Q.put(e, v),
```

```
get : Unit  $\rightsquigarrow$  write any E.Elem  
get() ≡  
  let (e, v') = Q.get(v) in v := v' ; e end  
  pre ~ is_empty(),  
  
is_empty : Unit → read any Bool  
is_empty() ≡ Q.is_empty(v)  
end
```

## Imperative sequential system

```
scheme I.MESSAGE1 =  
  hide IPQ, IQ in class  
  object  
    IPQ : I.PRI_QUEUE(T{Message for Elem}, T),  
    IQ : I.QUEUE(T{Message for Elem})  
  value  
    put : T.Message → write any Unit  
    put(m) ≡ IQ.put(m),  
  
    get : Unit ↗ write any T.Message  
    get() ≡ IPQ.get() pre can_get(),
```

```
can_get : Unit → read any Bool  
can_get() ≡ ~ IPQ.is_empty(),  
  
shift : Nat → write any Unit  
shift(n) ≡  
  if n = 0 ∨ IQ.is_empty() then skip  
  else  
    let m = IQ.get() in  
      IPQ.put(m) ; shift(n - 1) end end  
end
```

## Imperative to concurrent

- allows concurrent function calls without interference
- module by module: preserves structure
- “leaf” modules will have imperative state and “server” processes
- transformation: correct *by construction*

## Partial functions

We need to make interface functions total. *get* needs to return either a message or a result “no messages”.

```
scheme  
ELEM_RES =  
  extend ELEM with  
    class type Result == nil | result(elem : Elem) end
```

## Concurrent queue

```
scheme C_QUEUE(E : ELEM_RES) =  
  hide I, CH in class  
    object  
      I : I_QUEUE(E),  
      CH :  
        class  
          channel  
            empty : Unit,  
            put : E.Elem,  
            get : E.Result,  
            is_empty : Bool  
        end  
    end
```

## The server process

```
init : Unit → write any in any out any Unit  
init() ≡ I.empty() ; main(),  
  
main : Unit → write any in any out any Unit  
main() ≡  
  while true do  
    CH.empty? ; I.empty()  
    []  
    CH.get ! if ~ I.is_empty() then E.result(I.get()) else E.nil end  
    []  
    let e = CH.put? in I.put(e) end  
    []  
    CH.is_empty ! I.is_empty()  
  end,
```

## The interface processes

```
empty : Unit → out any Unit  
empty() ≡ CH.empty ! (),  
  
get : Unit → in any E.Result  
get() ≡ CH.get?,  
  
put : E.Elem → out any Unit  
put(e) ≡ CH.put ! e,  
  
is_empty : Unit → in any Bool  
is_empty() ≡ CH.is_empty?  
  
end
```

## Concurrent system

```
scheme C_MESSAGE1 =  
  hide PQ, Q, shift in class  
    object  
      PQ : C_PRI_QUEUE(T1{Message for Elem}, T1),  
      Q : C_QUEUE(T1{Message for Elem})  
  
    value  
      init : Unit → write any in any out any Unit  
      init() ≡ PQ.init() || Q.init() || shift(),
```

## Consequences of design paradigm

- States of leaf modules are independent: no interference.
- Interface functions of different leaf modules may be called sequentially or concurrently.
- Freedom from deadlock easy to check:
  - all channels hidden
  - all servers started by top-level *init*
  - interface functions and servers match.
- Emphasis on system design: some modules will be assumptions about the software and/or hardware environment.

```

put : T1.Message → in any out any Unit
put(m) ≡ Q.put(m),

get : Unit → in any out any T1.Result
get() ≡ PQ.get(),

can_get : Unit → in any out any Bool
can_get() ≡ ~ PQ.is_empty(),

shift : Unit → in any out any Unit
shift() ≡
    while true do
        case Q.get() of T1.nil → skip, T1.result(m) → PQ.put(m) end
    end
end
    
```

## Adding time

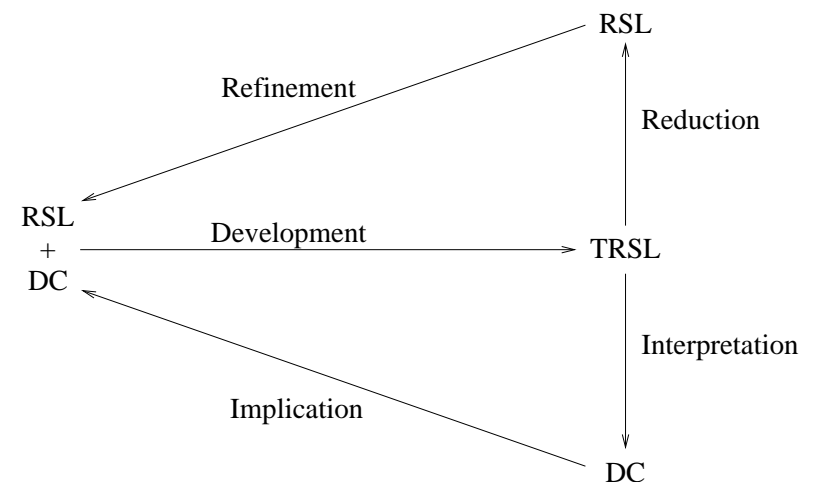
Timed RSL essentially just the addition of a **wait** expression.

```

value
    δ : Time • δ > 0.0,

shift : Unit → in any out any Unit
shift() ≡
    while true do
        wait δ ;
        case Q.get() of T1.nil → skip, E.result(m) → PQ.put(m) end
    end
end
    
```

## Timed development



## Timer variables

- may be started (set to zero) or reset (set negative)
- if not negative, are always incremented by **waits**
- measure durations

## An alarm

The alarm system may be enabled or disabled. When enabled, if it is disturbed, an alarm sounds. When disabled, disturbances are ignored.

The timing requirements are that after being enabled there is a period T1 before a disturbance causes an alarm. When it is enabled and there is a disturbance, there is a period T2 before the alarm sounds; if the system is disabled within this time there is no alarm.

## Untimed server

```
while true do
  enable? ; I.enable()
  []
  disable? ; I.disable()
  []
  disturb? ;
  if I.state() = enabled then (I.disturb() [] skip) end
end
```

## Timed server

```
while true do
  enable? ; I.enable() ; since_disturb := reset ; since_enable := 0.0
  []
  disable? ; I.disable() ; since_disturb := reset ; since_enable := reset
  []
  disturb? ;
  if I.state() = enabled ^ since_enable ≥ T1 ^ since_disturb ≤ 0.0
  then since_disturb := 0.0 end
  []
  delay() ;
  if since_disturb ≥ T2 then I.disturb() end
end
```

delay : Unit → write any Unit

delay() ≡

wait  $\delta$  ;

if since\_enable  $\geq$  0.0

then since\_enable := since\_enable +  $\delta$

end ;

if since\_disturb  $\geq$  0.0

then since\_disturb := since\_disturb +  $\delta$

end

## Conclusions

- wide spectrum, modular language
- effective method
- good documentation
- robust tools



## Developing a National Financial Information System

Trần Mai Liên, Lê Linh Chi, Nguyễn Lê Thu,  
Đỗ Tiến Dũng, Phùng Phương Nam, Hoàng Xuân Huấn,  
and Chris George



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2

### National Financial Information System

- Taxation
- Treasury
- Budget
- Spending ministries
- External loans and aid

- Large project
- Extensive introduction of computers
- Previous development uneven
- Customer has limited technical knowledge and capacity
- Requirements changing:
  - New kinds of tax
  - New accounting rules

Prime candidate for failure.

Trần Mai Liên, Lê Linh Chi, Nguyễn Lê Thu, Đỗ Tiến Dũng, Phùng Phương Nam, Hoàng Xuân Huấn, and Chris George

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## The gap

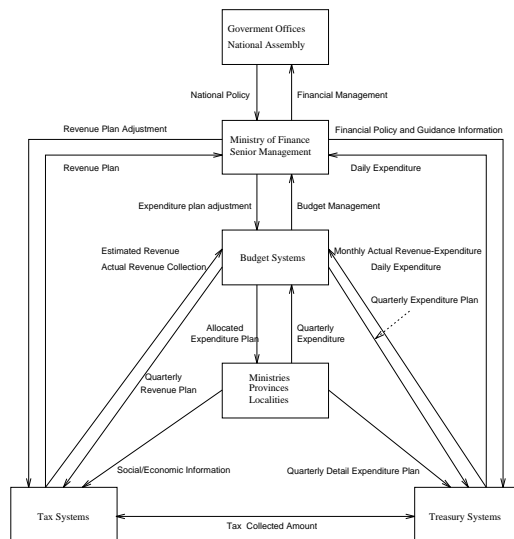
1. We need a national financial information system to collect reliable data, make budgets, assess the affects of possible changes, etc.
2. A small local office will need 8 PCs, ...

## Aim of the specification

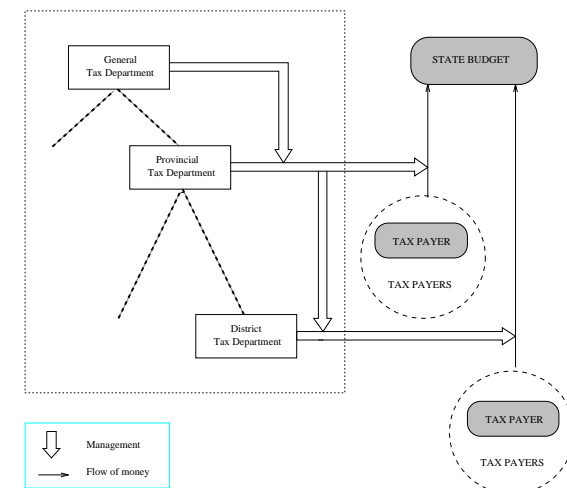
- Act as a high level design: allow design decisions to be explored.
- Define responsibilities for data storage, collection, analysis, reporting etc.
- and so provide detailed requirements for component offices.

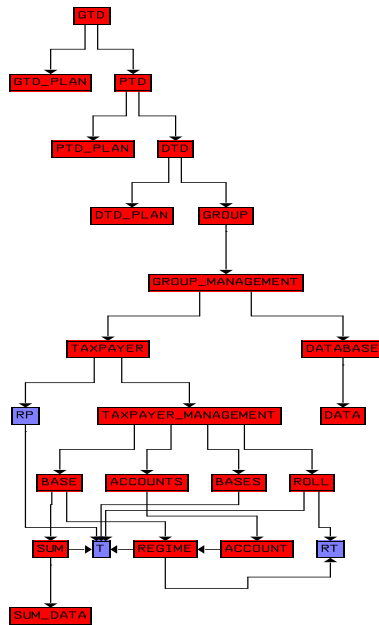
Not intended to be directly implemented! But provides a framework for gradual computerisation.

## MoF data flow diagram



## Tax system





## Order of development

- Tax accounting
    - Rapid prototype (via translation) and testing
  - Report generation and summarising
  - Tax system
  - Budget and treasury systems
  - External loans and aid systems
- Separate, parallel study on future of tax system.

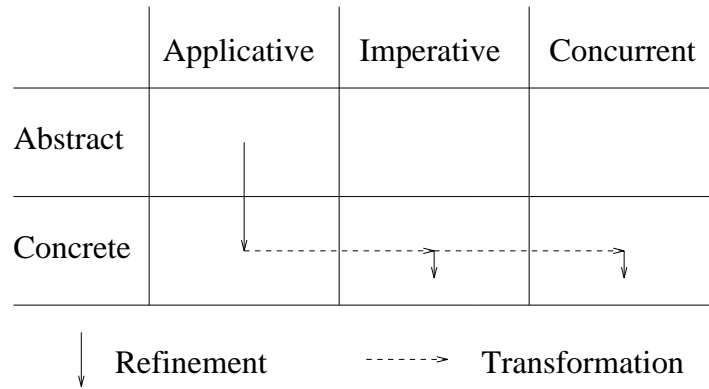
## Specification styles

- Applicative
  - convenient for specification
  - convenient for proofs
- Imperative sequential
  - convenient for implementation
  - twice as hard for proofs
- Imperative concurrent
  - convenient for implementation
  - 5 – 10 times as hard for proofs

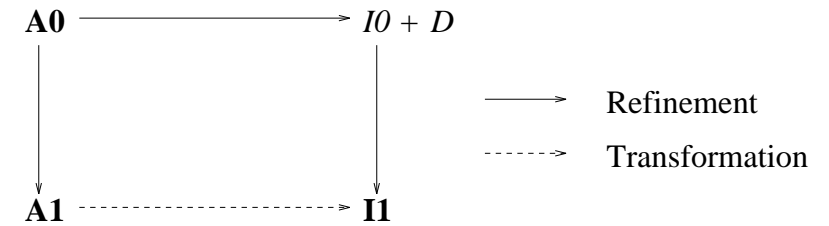
## Abstract axiom styles

- Applicative
  - $\text{is\_empty}(\text{empty}) \equiv \text{true}$
- Imperative sequential
  - $\text{empty}() ; \text{is\_empty}() \equiv \text{empty}() ; \text{true}$
- Imperative concurrent
  - $\forall \text{test} : \text{Bool} \rightsquigarrow \text{Unit} \bullet$   
 $(\text{main}() \# \text{empty}()) \# \text{test}(\text{is\_empty}()) \equiv$   
 $(\text{main}() \# \text{empty}()) \# \text{test}(\text{true})$

## Ideal development route



## Transformation theorem



### Theorem:

If  $\mathbf{A1} \preceq \mathbf{A0}$  and  $\mathbf{A1}$  is transformed to  $\mathbf{I1}$  then  
 $\exists$  module  $\mathbf{I0}$ , definitions  $D$  •

- $\mathbf{I1} \preceq \mathbf{I0}$
- $D$  conservatively extends  $\mathbf{I0}$
- **extend  $\mathbf{I0}$  with  $D \preceq \mathbf{A0}$**

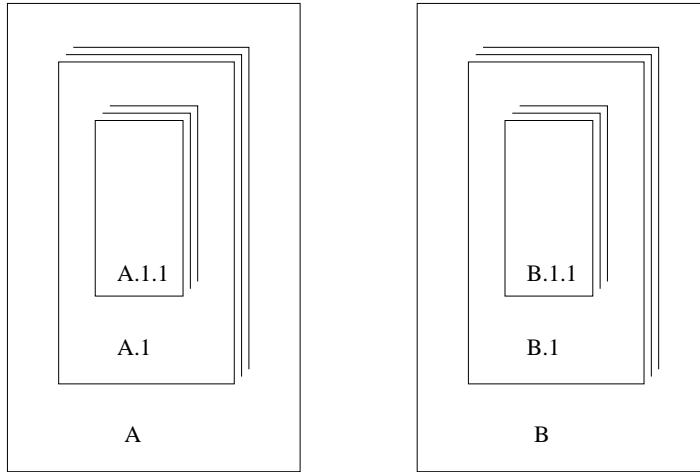
## Transformation properties

- could be automated (?); amenable to quality assurance
- applied compositionally on modules

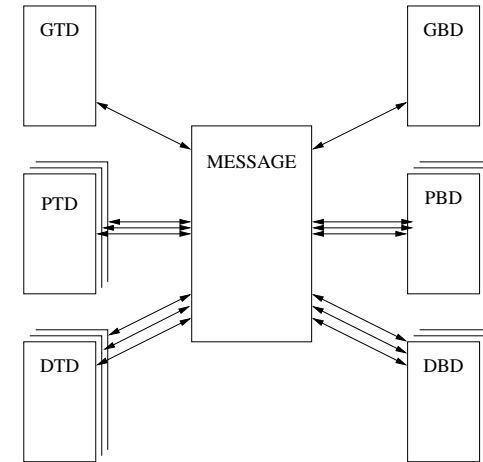
## Consequences

- deadlock freedom guaranteed
- for trees:
  - imperative state only in leaf modules; their states are independent
  - functions of lower level modules may be called sequentially or concurrently
- for acyclic graphs: calling sequences need more care

## Separate hierarchical subsystems



## Distributed and combined subsystems



## Hierarchical to distributed

Should be a *transformation*: reliable and repeatable.

- Preserve properties already checked
- Use standard modules
- Restrict editing to regular changes, i.e. with a pattern that can be checked

## Generic modules

- MESSAGE system for accepting and delivering messages
- CODE for transforming messages between global and subsystem types
- IN\_TRAY for receiving and storing messages
- SECRETARY for filling IN\_TRAY and, perhaps, dealing with some messages
- COUNTER for generating (locally unique) message numbers

Each office has instances of the last four, plus stub modules to replace lower-level office module.

## Stub modules

For each function in lower module called by upper:  
define a function of the same name and type to

1. get a new message number
2. code and send message to lower module
3. collect message with this number from in-tray
4. decode and return data from message

Provided communication works, the stub function behaves just like the original function.

## Conclusions

- Metatheorem that semantics of hierarchical calls are preserved (almost)
- Reliable transformational method supported by metatheory gives “correctness by construction”; proofs are avoided.

## References

- [1] Do Tien Dung, Le Linh Chi, Nguyen Le Thu, Phung Phuong Nam, Tran Mai Lien, and Chris George. Developing a Financial Information System. Technical Report 81, UNU-IIST, P.O.Box 3058, Macau, September 1996.
- [2] Do Tien Dung, Chris George, Hoang Xuan Huan, and Phung Phuong Nam. A Financial Information System. Technical Report 115, UNU-IIST, P.O.Box 3058, Macau, July 1997. Partly published in *Requirements Targeting Software and Systems Engineering*, LNCS 1526, Springer-Verlag, 1998.